



MATHEMATICS SPECIALIST : UNITS 1 & 2, 2023

Tuesday 8th August (T3W4)

Test 3 – Scalar Product and Trigonometry (10%)

1.3.10 – 1.3.14; 1.1.16; 2.1.1 – 2.1.9

Time Allowed 25 minutes	First Name	Surname	Marks / 24 marks
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Circle your Teacher's Name:

Ms Chua

Mr Koulianos

Mr Liu

Ms Robinson

Assessment Conditions: (N.B. Sufficient working out must be shown to gain full marks)

- ❖ Calculators: Not Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

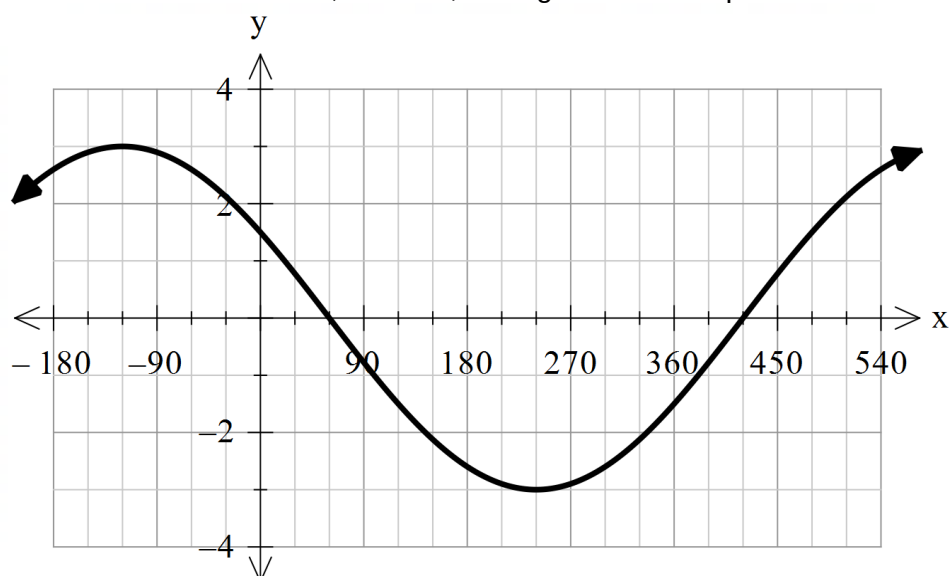
PART A – CALCULATOR FREE

Question 1 (2 marks)

Consider the vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$. Prove that $\mathbf{a} + \mathbf{b}$ is not perpendicular to $\mathbf{c}$.

Question 2 (3 marks)

The equation of the graph below is in the form $y = -a \sin[b(x^\circ - c^\circ)]$, where a, b and c are positive. Determine the values of a, b and c , stating the smallest possible value for c .



Question 3 (6 marks = 2, 2, 2)

Vectors x and y are such that $|x| = 5$, $|y| = 4$ and $x \cdot y = 10\sqrt{3}$.

a) Determine the angle between x and y .

b) Determine the scalar projection of y onto x .

c) Determine $(x + 2y) \cdot (x - y)$.

Question 4 (7 marks = 4, 3)

a) Solve $\sin 3x - \sin x = \cos 2x$ for $0 \leq x \leq 2\pi$.

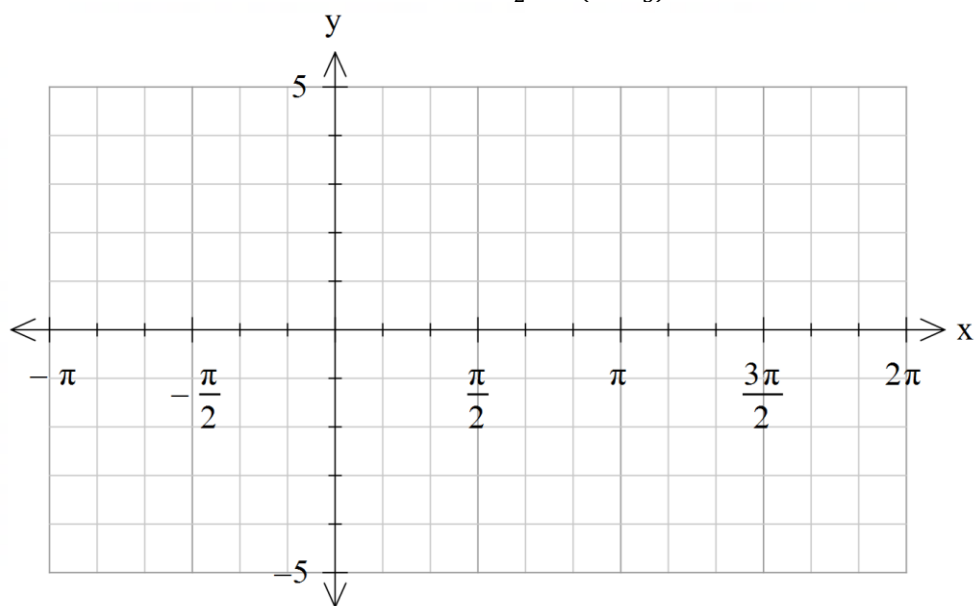
b) In terms of $n \in \mathbb{Z}$, state the general solution to $2 \sin^2 x^\circ = \sin x^\circ + 1$.

Question 5 (3 marks)

In terms of $n \in \mathbb{Z}$, state the general solution to $2 \sec^2 4\theta + \tan^2 4\theta = 3$, where θ is in radians.

Question 6 (3 marks)

On the axes below, sketch the graph of $y = \frac{1}{2} \sec\left(x + \frac{\pi}{3}\right) - 1$, showing clearly all key features.





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PART B – CALCULATOR ASSUMED

Question 7 (5 marks = 3, 2)

- a) Express $2 \cos \theta - 3 \sin \theta$ in the form $r \cos(\theta + \alpha)$ with exact r , and acute α in radians, correct to two decimal places.

- b) Hence, determine the minimum value of $2 \cos \theta - 3 \sin \theta$ and the first positive value of θ for which it occurs.

Question 8 (3 marks)

Consider the vectors $x = \begin{pmatrix} a \\ b \end{pmatrix}$, $y = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $z = \begin{pmatrix} 14 \\ -5 \end{pmatrix}$.

Determine the values of a and b given that x and y are perpendicular, and $x + z$ is parallel to y .

Question 9 (7 marks = 3, 4)

Prove each of the following.

a) $\frac{\cos t}{1 - \sin t} = \frac{1 + \sin t}{\cos t}$

b) $\frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t} = \tan t$

Question 10 (4 marks)

Prove $\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$.

Question 11 (5 marks = 1, 1, 3)

Shohei is on holiday and takes a ride on a Ferris wheel. He uses his smart phone to measure his altitude during the ride. The equation $H = -24.5 \cos\left(\frac{\pi}{5}t\right) + 26$ can be used to model the data collected, where H is the height in metres above ground level and t is the time in minutes since the ride commenced.

- a) How long does it take to complete one revolution on the Ferris wheel?

- b) What is the diameter of the Ferris wheel?

- c) The ocean is not far from where the Ferris wheel is located but can only be seen from a height of at least 40 metres above the ground. For what percentage of the ride will Shohei be able to see the ocean? Give your answer correct to two decimal places.